Holographic Discreteness of Inflationary Perturbations

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The holographic entropy bound is used to estimate the quantum-gravitational discreteness of inflationary perturbations. In the context of scalar inflaton perturbations produced during standard slow-roll inflation, but assuming that horizon-scale perturbations "freeze out" in discrete steps separated by one bit of total observable entropy, it is shown that the Hilbert space of a typical horizon-scale inflaton perturbation is equivalent to that of about 10^5 binary spins— approximately the inverse of the final scalar metric perturbation amplitude, independent of other parameters. Holography thus suggests that in a broad class of fundamental theories, inflationary perturbations carry a limited amount of information (about 10^5 bits per mode) and should therefore display discreteness not predicted by the standard field theory. Some manifestations of this discreteness may be observable in cosmic background anisotropy.

I. INTRODUCTION

The origin of cosmological perturbations now appears to be well understood from the quantum theory of fields in curved spacetime [1–7]. They originate during inflation as zero point fluctuations of the quantum modes of various fields— the inflaton giving rise to scalar perturbations, the graviton to tensor perturbations. The field quanta in the original fluctuations convert to classical perturbations as they pass through the de Sitter-like event horizon; they are then parametrically amplified by an exponential factor during the many subsequent e-foldings of inflation, creating an enormous number of coherent quanta in phase with the original quantum seed perturbation. From the classical point of view, the quantum fluctuations create perturbations in the classical gravitational gauge-invariant potential ϕ_m [8], leading to observable background anisotropy and large scale structure [9–16]. All stages of this process are under good calculational control, even the conversion of quantum to classical regimes [17–22]. The phase and amplitude of the large-scale classical perturbation modes observed today are a direct result of the quantum field activity during inflation; indeed, the pattern of microwave anisotropy on the largest scales corresponds to a faithfully amplified image of microscopic field configurations as they froze out during inflation. Roughly speaking, each hot or cold patch on the sky derives originally from about one quantum.

The standard calculation of these processes [23,24] uses a semiclassical approximation: spacetime is assumed to be classical (not quantized), and the perturbed fields (the inflaton and graviton) are described using relativistic quantum field theory, essentially (in the limit of free massless fields) an infinite collection of quantized harmonic oscillators. The Hilbert space of this system is infinite, so although the fields are quantized, they are continuously variable functions that can assume any values. The creation of the particles can be viewed as an effect of the nonadiabatic expansion, and the "collapse of the wavefunction" (in this case, "freezing out") can be described as a unitary quantum process of state squeezing. The theory generically predicts random-phase gaussian noise with a continuous spectrum determined by the parameters of the inflaton potential. In this approximation, there is no telltale signature of quantum discreteness in the observable classical remnant— the anisotropy of the background radiation. Although sky maps contain images of "single quanta," their spectrum is continuous and the amount of information is in principle infinite.

It has always been acknowledged that this description is incomplete, and will be modified by including a proper account of spacetime quantization. Although the fundamental theory of such quantization is not known, a "holographic entropy bound" already constrains with remarkable precision the total number of fundamental quantum degrees of freedom. The complete Hilbert space of a bounded volume is finite and discrete rather than infinite and continuous, limiting the range of accessible configurations in any region to a definite, calculable number. In particular this limit applies to inflaton quanta collapsing into classical metric perturbations. This paper uses the holographic entropy bound to estimate where field theory breaks down in the inflationary analysis, the effective dimension of the Hilbert space for the observable perturbations when they freeze out, the maximum amount of information contained in the perturbations, and the level at which quantum-gravitational discreteness appears in cosmic background anisotropy. The main result is that in fundamental theories where the holographic entropy bound arises from discrete fundamental eigenstates, the amount of information in the anisotropy is remarkably limited: it can be described with only about 10^5 bits per sky-harmonic mode, implying that the perturbations should be pixelated in some way. In principle, this effect may be observable, and provide concrete data on the discrete elements or eigenstates of quantum gravity.

II. HOLOGRAPHIC BOUND ON INFORMATION CONTENT

Quantitative insights into the Hilbert space of inflationary spacetimes come from the thermodynamics of black holes [25–32] and de Sitter spacetimes [33]. The black hole entropy derived from thermodynamic reasoning appears to be complete, in the sense that it includes all possible degrees of freedom of mass-energy that make up the hole. If we insist that black holes are quantum mechanical objects that obey unitary evolution, they must somehow encode all the information counted in this entropy, then radiate it to spatial infinity as Hawking quanta as they evaporate [34–37]. Such considerations led 't Hooft and Susskind to propose [38,39] a "holographic principle" [40] for physical systems: the total entropy S within any surface is bounded by one quarter of the area A of the surface in Planck units. (Unless otherwise indicated, throughout this paper we adopt $\hbar = c = G = 1$, with Planck mass $m_{Planck} = (\hbar c/G)^{1/2} = 1.22 \times 10^{19}$ GeV.) This is an "absolute" entropy; the dimension N of the Hilbert space is given by e^S, and the total number of distinguishable quantum states available to the system is given by a binary number with $n = S/\ln 2 = A/4\ln 2$ digits [41].

Bousso [42] has collected and reviewed these arguments, and proposed a more rigorous formulation called the "covariant entropy bound": the area of a surface gives a bound on total entropy, not of the enclosed spacelike 3-volume but of the 3-volume defined by certain lightsheets propagated from the surface. This bound is apparently universal in nature; at least, no counterexamples to it have been found. Although the bound seems to hold in all physical situations we can imagine, there does not seem to be any way to derive why this should be so, given only the physics we have. The fact that it always works presumably reflects a deep structure in quantum gravity.

Although a true derivation of the covariant entropy bound is not yet known, its seems to originate from a fundamental theory that incorporates discrete elements or eigenstates. Recent examples of theories that display this property explicitly include loop quantum gravity [43] (where discrete eigenstates appear at a very early stage and appear to be a fundamental feature) and M-theory, where it is demonstrated in particular situations where discrete degrees of freedom can be explicitly traced via holography (such as AdS5/CfT) to discrete symmetries of projective, dual field theories. These types of fundamental structures would also impose a logarithmically finite constraint on the number of possible solutions of fields during inflation— that is, jumps between possible configurations correspond to changes "in the exponent" of the number of states, in the same way that adding just one more discrete element (e.g., a spin-1/2 particle) to a system of n binary spins discontinuously grows the Hilbert space from dimension 2^n to 2^{n+1}.

This property is the most important assumption made in the quantitative estimates of observable discreteness of background radiation anisotropy derived below. It is probably general enough to apply across a broad class of theories, including those now receiving the most theoretical attention. Indeed, the main point is that anisotropy data may provide our first direct view of the underlying, basic elements or eigenstates of the fundamental theory.

The holographic conjecture has been analyzed in a variety of relativistic cosmologies, including de Sitter and anti-de Sitter space [44–50]. The cosmological version of the holographic bound is [51,52] that the "observable entropy"[1] of any universe cannot exceed $S_{max} = 3\pi/\Lambda$, where Λ is the cosmological constant in Planck units. In a de Sitter universe, as in a black hole, this corresponds to one quarter of the area of the event horizon in Planck units, but the bound is conjectured to hold for any spacetime, even Friedmann-Robertson-Walker (FRW) universes with matter as well as Λ. In the de Sitter case, which we will adopt as a local model for the inflationary spacetime, $S_{max} = \pi/H^2$, where H is the expansion rate.

The bound includes all degrees of freedom of all matter fields as well as all quantum degrees of freedom of the spacetime itself. It implies for example, that a number with $\pi/H^2 \ln 2$ bits is sufficient to specify everything that is going on in a causal diamond of an inflationary universe. We consider the constraints imposed by the entropy bound on a particular experiment conducted by nature during inflation: the formation of classical perturbations from quantum fluctuations— or equivalently, nonadiabatic particle production and phase wrapping of a highly squeezed field state.

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[1]The observable entropy corresponds to the information contained in a "causal diamond", a spacetime volume bounded by two intersecting light cones, one open in the future direction of the beginning of an experiment and one in the past direction of the end of the experiment.

III. INFORMATION CONSTRAINTS ON FIELD MODES DURING INFLATION

We adopt as an idealized model of the inflationary period a portion of de Sitter spacetime [51,53]. Figures 1 and 2 illustrate different views of de Sitter spacetime, each showing two different ways of laying down time and space coordinates. Usually, the field theory analysis of inflationary perturbations is done with the FRW-like slicing of de Sitter spacetime, since these space slices map directly onto the usual spacelike hypersurfaces of the postinflationary FRW metric. The metric in this slicing, which covers half of the full de Sitter solution, appears to have flat spatial slices:

$$ds^2 = -d\tau^2 + \exp[2H\tau]dx^i dx_i. \tag{1}$$

The comoving modes have fixed lengths in the Euclidean spatial coordinates x_i.

The Penrose diagram for a whole inflationary spacetime, including the FRW part at late times, is shown in figure (3). This diagram shows the route by which information flows and by which observable patterns are imprinted on the background radiation by quantum perturbations. For completeness, figure (3) includes not only the causal path for creating microwave background perturbations, but also the trajectories for gravitons in a possible high-frequency gravitational wave background, such as might be detected directly by interferometers such as LIGO, LISA or their successors at frequencies $\approx 10^{\pm 3}$ Hz. It can be produced by quantum graviton fluctuations during inflation, and also classically by mescoscopic mass motions at later times from e.g. symmetry breaking, dimensional reduction, or reheating. Holographic discreteness might manifest itself in the directly measured classical metric perturbation of the inflationary waves, but that effect is beyond the scope of this paper.

Another slicing of the de Sitter spacetime (which resembles an inside-out version of a black hole spacetime in Schwarzschild coordinates) is described by the metric

$$ds^2 = -U(r)dt^2 + U^{-1}(r)dr^2 + r^2 d\Omega_2^2, \tag{2}$$

where

$$U(r) = 1 - \frac{r^2}{r_0^2}, \tag{3}$$

and $d\Omega_2$ is the (2D) angular interval. The event horizon stands at radius $r_0 = H^{-1}$, the Bekenstein-Hawking temperature is $T_{BH} = (2\pi r_0)^{-1}$, and the event horizon area is

$$A = \frac{4\pi}{H^2} = \frac{12\pi}{\Lambda}. \tag{4}$$

These coordinates cover the entire "causal diamond" accessible to any actual observation by an observer at the origin, comprising a finite spatial region (as opposed to the infinite volume of the FRW slicing) subject to the holographic entropy bound. Figure 1 shows a spacetime embedding diagram of this volume with the two slicings in physical units, and figure 2 compares their Penrose diagrams.

It has been established that holography is not consistent with independent field modes [42], and this can be seen from considering the flow of information in these figures. A spacelike hypersurface in the coordinates (2) extends into the distant, highly redshifted past and gets very close to the event horizon. The bulk of the information from a typical small 3-volume near such a surface in the distant past ended up getting advected out of the horizon. (The full information of a 3-volume only survives from the Planck scale to the horizon scale for a single Planck-sized patch in the very center of the volume at $r = 0$). Following one of the hypersurfaces (2) back to where it lies within a Planck wavelength of the horizon corresponds to a redshift of only $\sqrt{2/H}$ relative to where it intersects the origin (rather than $\approx H^{-1}$ which takes a horizon-scale mode to the Planck length). Since this is the regime where we expect quantum-gravitational information mixing among modes, the vacua of the modes will be affected by quantum gravity at a physical mode wavelength much larger than the Planck scale. The "stretched event horizon" of the de Sitter space acts like the event horizon of a black hole, thermalizing the system by allowing (strong gravitational) interactions between modes of different wavelength, so that unitary evolution is not independent for each comoving-wavelength mode; instead their information is mixed.

Consider the holographic constraints on the propagation of physical influences from above the Planck scale. Note that for realistic inflation models with H well below the Planck scale, the maximum observable entropy $S_{max} = \pi/H^2$ is always much less than the 3-D de Sitter volume, $4\pi/3H^3$. This means that the holographic bound precludes information from the super-Planck regime from directly propagating to observable scales: there isn't enough information

available in the de Sitter volume to specify the state of every mode on the Planck scale at any given time. Conversely, when any given mode has expanded to the de Sitter scale, much less than one bit of information per de Sitter volume survives from "its own" Planck epoch. Holography allows at most on average only one bit per mode at a scale $\lambda_{hol} \approx (\lambda_{Planck}^2/H)^{1/3}$; below this scale holography tells us that it is inconsistent to assume that the states of each mode can be specified independently, because there is not enough information available to do so. (Incidentally, this scale is about a fermi today, based on the current estimates of Λ; however this has no effect on any local experiment.)

While the effects of super-Planckian physics have been a subject of debate within the field theory framework [54–59], these information-counting arguments suggests that in these models, *any* field theory omits an important constraint of spacetime quantization on the fluctuations even well below the Planck scale. Of course, counting arguments do not inform us by what mechanism the holographic constraint intervenes.

IV. DISCRETENESS OF COSMIC BACKGROUND ANISOTROPY

A. Minimal discreteness: spacetimes quantized in discrete steps of N

The holographic constraint clearly imposes some discreteness on the behavior of an inflationary spacetime. At the very least, if the classical H adjusts itself in such a way that the Hilbert space dimension N (as the fundamental quantity [60]) is to come out to be an integer, H proceeds through a series of discrete steps with

$$H_i = \sqrt{\frac{\pi}{\ln N_i}} \tag{5}$$

where N_i are integers. For example, nearly exact Planck-scale inflation $H = \sqrt{\pi/\ln(23)} = 1.000972\ldots$ occurs for $N = 23$.

In principle, this property is observable. The pattern of microwave anisotropy on the sky on large scales includes direct images, created by the Sachs-Wolfe effect, of the inflaton perturbations in about $\ell_{rec}^2 \approx 10^4$ independent de Sitter volumes, where ℓ_{rec} is the angular wavenumber corresponding to the horizon at last scattering. (Below this scale the observed anisotropy is strongly affected by plasma oscillations and propagating modes, so the primordial information is less directly visible). Thus in this example we can observe 10^4 samplings of a process that originally only yielded 23 distinguishable outcomes, and we should be able to discern repeated patterns.

Of course, we cannot yet predict what those patterns are, since we don't know what the states of quantum gravity look like (although Bekenstein [41] has made quantitative conjectures about quantum states of black hole spacetimes). The quantum gravitational discreteness could manifest itself in a very simple way, such as a discrete component to the spectrum of spherical harmonics instead of the continuous gaussian random phase distribution predicted by the field theory approximation; or in a more subtle way, as repeated patterns of sky pixel amplitudes. In this case the discreteness may be more conspicuous in the phase information (as opposed to the power spectrum amplitude) of anisotropy.

The number of distinguishable states grows exponentially, $N = \exp[\pi H^{-2}]$. If this corresponds to the number of possible options for the horizon-scale fluctuations in a de Sitter spacetime, the holographic discreteness is only observable if H lies within a factor of a few of the Planck scale. Even in principle, the largest number of independent inflationary horizons on the sky (observable in principle in an inflationary gravitational-wave background) is "only" 10^{58} (the inverse solid angle of a comoving Planck patch); therefore if $\pi/H^2 \ln(10) \geq 58$, the discrete system becomes indistinguishable from a continuum. Thus we certainly need $H \geq 0.15$ for an observable effect.

We know that H during inflation is not nearly this large. Quantum graviton fluctuations lead to classical tensor-mode perturbations of amplitude $h_{rms} \approx H$. This leads to a general upper bound (discussed in more detail below) on the Hubble constant during inflation, from observations of large angular scale background anisotropy, of about $H \leq 10^{-5}$. The maximum observable de Sitter entropy exceeds $S_{max} \approx 10^{10}$, which means that the total number of quantum states exceeds $\exp[10^{10}]$. If the number of distinguishable states accessible to an inflaton mode when its wavefunction collapses were this large, the usual continuum approximation would be a good one for all conceivable microwave background observations on large scales.

However, this estimate does not apply in many theories. Holographic discreteness in string theory and loop quantum gravity, like discreteness in the Hilbert spaces of more familiar quantum systems such as collections of spinning particles, probably derives ultimately from discrete elements in the fundamental theory. In the remainder of this paper we assume that it is these elements, and not the states, which can only be added "one at a time". In this case, fluctuations are quantized in discrete steps of n (or S) rather than N.

B. Fluctuations quantized in discrete steps of S

Holography tells us that the maximum observable total entropy and the de Sitter expansion rate are connected by $S_{max} = \pi/H^2$. We assume that this connection arises from fundamental discrete elements so that S (or $n = S/\ln 2$) occurs in integer steps, fixing a finite set of discrete values for H. Scalar perturbations, which appear today as observable temperature perturbations, are determined classically by the dynamics of the inflaton field ϕ; the inflaton perturbations originate as quantum states, and when they freeze, they in turn directly fix H, so that their configurations are also constrained to a finite discrete set of options. We now proceed to estimate observable discreteness— characterized by the number of options available, or the amount of information contained in the classical scalar perturbations— following these connections.

To estimate the graininess associated with holographic constraints on inflaton fluctuations, we consider a very simple toy model, where observable regions of quasi-de Sitter spaces occur in a discrete set of eigenstates of H with splitting ΔH between adjacent levels:

$$\ldots \leftrightarrow |H_0 + \Delta H > \leftrightarrow |H_0 > \leftrightarrow |H_0 - \Delta H > \leftrightarrow |H_0 - 2\Delta H > \leftrightarrow |H_0 - 3\Delta H > \leftrightarrow \ldots, \tag{6}$$

where $|H_0>$ denotes a reference de Sitter space of with expansion rate H_0. The secular classical evolution of an inflationary spacetime represents a steady rightward flow, and quantum fluctuations can go in either direction (although the field-theory fluctuations are in general coherent over many of these transitions). This leads to discreteness in observable anisotropy with steps of some amplitude $\Delta T/T$.

The key assumption of the toy model is that *the background spacetime and horizon-scale fluctuations occur through a sequence of transitions between discrete states, each of which adds (or subtracts) one bit of information to the total maximum observable entropy.* That is, instead of the H coming in discrete steps of N as in Eq. (5), it comes in discrete steps of n:

$$H_i = \sqrt{\frac{\pi}{n_i \ln 2}} \tag{7}$$

where n_i are integers. Once we have assumed discrete steps in H, this *ansatz* represents a plausible lower bound on the step size required for consistency: if ΔH were smaller, the total entropy increment (in all modes of all fields) would be less than one bit, which is not enough even to include the degree of freedom represented just by the jump in H.

The final state is the product of transition operators, the final information the sum of that in the transitions. The change in the dimension of the total Hilbert space for the cumulative effect— including the change in the set of all possible field fluctuations— is still very large, $\delta N = \exp[S_{max}]\delta S_{max}$; this reflects the cumulative effect of choices made in each new bit. The holographic bound indicates that the Hilbert space is the same size as a collection of n spins; in this model, spacetimes are assumed to exhibit the same discreteness as a discrete integer number of such spins.

To estimate the observable consequences of the toy model, we use the classical dynamics of the inflaton, the bulk of which forms a nearly homogeneous zero-momentum condensate. The classical expectation value ϕ_c of the inflaton field has a unique connection to the entropy through its effect on the spacetime geometry: the effective potential $V(\phi_c)$ and de Sitter rate H that controls the total entropy are related via the Friedmann equation,

$$H^2 = \frac{8\pi}{3}\left[V(\phi_c) + \dot{\phi}_c^2/2\right]. \tag{8}$$

Therefore, large-scale classical perturbations in the inflaton field are directly mapped onto total observable entropy and also, in the toy model, come in discrete steps. We assume that in the discrete sequence of de Sitter eigenstates, eigenstates of H are also eigenstates of ϕ_c.

The background (unperturbed, continuous, classical) evolution of a homogeneous noninteracting scalar field ϕ_c, is controlled by the classical dynamical equation [24,61]

$$\ddot{\phi}_c + 3H\dot{\phi}_c + V'(\phi_c) = 0, \tag{9}$$

where $V' \equiv dV/d\phi_c$. We assume for simplicity that the observed modes crossed the inflationary horizon during a standard so-called "slow roll" or Hubble-viscosity-limited phase of inflation, corresponding to $V'/V \leq \sqrt{48\pi}$ and $V''/V \leq 24\pi$, during which $\dot{\phi}_c \approx -V'/3H$. The rate of the roll is much slower than the expansion rate H, so the

kinetic term in Eq. (8) can be ignored in the mean evolution. The slow-roll phase of inflation creates approximately scale-invariant curvature perturbations.

We define a combination of inflationary parameters by

$$Q_S \equiv \frac{H^3}{|V'|} \approx \left(\frac{V^3}{V'^2}\right)^{1/2} \left(\frac{8\pi}{3}\right)^{3/2}. \tag{10}$$

This combination controls the Hilbert space attached to the perturbations. It also happens that it can be estimated fairly accurately, since it also controls the amplitude of the scalar perturbations observed in the microwave background anisotropy. Adopting the notation of [62], the anisotropy can be decomposed into scalar and tensor components,

$$\frac{\langle Q\rangle^2}{T_\gamma^2} = S_Q + T_Q, \tag{11}$$

where $\langle Q\rangle$ denotes the (global) mean quadrupole anisotropy amplitude, $T_\gamma = 2.725 \pm 0.002$ the mean temperature [62], and the scalar and tensor contributions S_Q and T_Q to the mean quadrupole C_2 are given by [61]

$$S_Q = \frac{5C_2^S}{4\pi} \approx 2.9\frac{V^3}{V'^2} \tag{12}$$

and

$$T_Q = \frac{5C_2^T}{4\pi} \approx 0.56V, \tag{13}$$

using numerical factors for a slow-roll, flat Λ model. The best fit to the four-year COBE/DMR data, assuming scale invariance ($n = 1$, and $T_Q = 0$), yields [10,11] $\langle Q\rangle = 18 \pm 1.6\mu$K, and hence combining the above,

$$Q_S = 9.4 \pm 0.84 \times 10^{-5}. \tag{14}$$

(Note that if we instead assume $S_Q = 0$, these formulae also yield the upper bound $H \leq 2\times 10^{-5}$ discussed above, from the tensor modes alone.)

During slow-roll inflation, there is a steady increase in observable entropy at a rate

$$\dot{S}_{max} = \frac{8\pi^2}{9}\frac{V'^2}{H^5} = 8\pi^2 H Q_S^{-2}. \tag{15}$$

Every inflationary e-folding, S_{max} increases by an amount of order 10^{10} due just to the classical evolution of the system. (In a time $\approx 10^{-10}/H$, the total observable entropy changes by about one bit.) The important point, which we will now elaborate, is that the information attached to the observable quantity— the horizon-scale perturbation in the inflaton— is much smaller than this, of order 10^5. The basic reason is that the horizon-scale perturbations contribute only a small fraction $\approx 10^{-5}$ of the total change in observable entropy; consequently, we conjecture that a typical perturbation can be completely characterized by $\approx 10^5$ bits per Hubble volume. The framework for this argument is illustrated in figure (4).

Consider a spatially uniform change $\delta\phi_c$ in the classical inflaton condensate, extending over a volume greater than $(4\pi/3)H^{-3}$. This changes the total observable de Sitter entropy in the affected volume by an amount

$$\delta S_{tot} = \frac{-2\pi}{H^3}\delta H = \frac{-2\pi}{H^3}\frac{4\pi}{3H}V'\delta\phi_c. \tag{16}$$

Now consider the effect of a quantum perturbation in the inflaton field of amplitude $\delta\phi$, assuming that it behaves like a "frozen in" classical spacetime background. Substituting from the classical slow-roll formulae above, an estimate for the jump in total entropy associated with a horizon-scale perturbation $\delta\phi$ can be directly expressed in terms of the observable quantity Q_S,

$$\delta S_{tot} = \frac{8\pi^2}{3}\left[\frac{\delta\phi}{H}\right] Q_S^{-1}. \tag{17}$$

The standard field theory analysis for the horizon-size perturbations predicts that as they are frozen in (or squeezed) into a classical state, the quantity $[\delta\phi/H]$ is statistically determined, with a continuous gaussian statistical distribution

of order unit width. The corresponding increment $\delta S_{qft} \approx (8\pi^2/3)Q_S^{-1}$ then is roughly the jump in the total observable cosmological entropy associated with the creation or destruction of a single horizon-scale inflaton quantum.

Therefore, the standard quantum transition corresponding to a typical perturbation yields a total change of no more than about $\delta S_{qft} \approx (8\pi^2/3)Q_S^{-1} \approx 10^5$ *in the maximum observable entropy.* This is much less than the increase given by Eq. (15) in the total information during an expansion time. Thus the amount of information δS_{qft} attached to the perturbation in the horizon-size inflaton quantum $\delta\phi$ as it freezes into a classical state is much less than the total growth of information of the spacetime during the same time, of order Q_S^{-1} rather than Q_S^{-2}. This may be enough to produce a detectable level of discreteness.

Is this story consistent with the more general ideas that motivated holography in the first place? The naiive assumption in our toy model is that the dimension of the observable Hilbert space changes in steps of one bit in the exponent, with each binary choice corresponding to a transition to a new quasi- de Sitter eigenstate. This does not mesh trivially with the idea of preserving overall unitary evolution so this cannot be quite the whole story; but indeed it is not a trivial matter to even define unitarity rigorously in deSitter spacetime [50]. Another comment is that modes much smaller than the horizon have their accessible Hilbert space changed due to a quantum fluctuation on the horizon scale. From the point of view of an observer at rest in the center of a "$\delta\phi$ patch," much of the incremental information attached to $\delta\phi$ appears in modes much smaller than H^{-1}— the short-wavelength modes close to the stretched event horizon that are still having their vacuum initial states formed, and only later expand to the horizon scale. Much of the "new" information represented by δS_{tot} must be available to modes near the stretched event horizon as viewed by any given observer— modes which are much smaller than H^{-1}. Thus it makes sense that the information available to the configurations of horizon-scale fluctuations is much smaller than S_{max}.

C. Toy Model of Anisotropy with Discrete Levels of T

It is useful to delve into some more detail with a more concrete, albeit unrealistic, toy model for discreteness in the perturbations. Suppose that the possible values of the horizon-size inflaton perturbation $\delta\phi$ are selected from a discrete set of levels separated by some step size $\Delta\phi$. The incremental observable information made available by the transition between levels is

$$\Delta S_{tot} = \frac{8\pi^2}{3}\left[\frac{\Delta\phi}{H}\right] Q_S^{-1}, \tag{18}$$

including all the degrees of freedom of all the fields, not just the inflaton. We now assert that *jumps in* $\delta\phi$ *occur in steps of at least a certain minimum size* $\Delta\phi$*, such that* $\Delta S_{tot} \geq \ln 2$ *(that is, a change of at least one bit in total observable entropy).* This leads to a definite minimum step size,

$$\left[\frac{\Delta\phi}{H}\right] \geq Q_S(3\ln 2/8\pi^2) = 2.5 \times 10^{-6}. \tag{19}$$

As above, this *ansatz* seems necessary for consistency: a step at least this large is needed just to specify the degree of freedom represented by the $\Delta\phi$ transition itself.

Taking this model literally and assuming subsequent linear classical evolution, this discreteness of the field amplitudes leads to a discrete set of values for the curvature perturbation (e.g., Bardeen's [8] ϕ_m), and therefore the observed temperature perturbation δT, which is largely determined by this quantity via the Sachs-Wolfe effect. The discrete jumps are smaller than the total standard predicted perturbation amplitude by about the factor

$$\frac{\Delta T}{\delta T} \approx \frac{\Delta\phi}{\delta\phi}. \tag{20}$$

That is, the amplitude of the anisotropy on the sky, instead of coming from a continuous distribution (with variance $\approx \langle Q\rangle^2$ contributed by fluctuations in each octave of angular wavenumber), comes from a discrete distribution, with values of T formed by a sum of discrete increments of order ΔT, where

$$\left[\frac{\Delta T}{\langle Q\rangle}\right] \approx \left[\frac{\Delta\phi}{H}\right]. \tag{21}$$

The amplitude difference at which the discreteness appears is about

$$\Delta T \geq \left[\frac{\Delta\phi}{H}\right]\langle Q\rangle \approx 10^{-10}\ \mathrm{K}. \tag{22}$$

V. OBSERVATIONS?

The toy-model results should not be taken as a literal physical prediction of the character of the discreteness; such a prediction would require understanding of the true eigenmodes of the system. We do not know from these arguments what their detailed character is, or how they might be manifested in the $A_{\ell m}$ spectrum of the anisotropy. (For this reason, we have also glossed over the facts that observed anisotropy is a superposition of modes from different comoving scales, and that these are increasingly affected, on scales smaller than the quadrupole, by propagation effects before, during and after recombination that modify the primordial signal [16]).

The analysis above does however highlight several new and unexpected features that might apply to the real-world discreteness: (1) The characteristic fractional amplitude of the inflaton perturbation discreteness is not exponentially small, but is comparable to the overall scalar perturbation amplitude. (2) The information in a typical horizon-scale perturbation does not depend explicitly on unknown parameters such as the values of H or V', except through the observable combination Q_S. (3) *The holographic counting arguments suggest that an inflaton perturbation at the time it freezes out has a Hilbert space equivalent to only about* 10^5 *binary spins.* We therefore conjecture that the theory incorporating the true eigenmodes may predict some (unspecified) kind of discreteness corresponding to less than about 10^5 bits of information per mode.

The observability of discreteness depends greatly on how it is manifested. To choose a somewhat silly example, suppose that rather than occuring in $\approx 10^5$ discrete levels of amplitude, a perturbation is composed of $\approx 10^5$ discrete binary pixels on the sky. This level of pixelation in the quadrupole modes (say) would not be completely erased by transport effects at recombination, which propagate on an angular scale comparable to this pixel size (for example, the angular wavenumber of the first "acoustic peak" in the angular power spectrum C_ℓ^2 is at $\ell \approx 200$, corresponding to $\ell^2 \approx 4 \times 10^4$ pixels). With discreteness encoded this way, each pixel might be either hot or cold, $\delta T/T = \pm 1 \times 10^{-5}$ (say). Intermediate gray levels would appear as a checkerboard pattern of alternating hot and cold pixels on a scale of about a degree of arc. Such a pattern would be very conspicuous in the current data, even allowing for the complication that it would be superimposed on approximately gaussian noise from smaller scale modes of comparable overall amplitude.

Thus, for some kinds of eigenmodes, observations of holographic discretness may be practical. The possibility of finding such a qualitatively new effect motivates a search in the microwave background data for discrete behavior, in either the amplitudes or the multipole phases. Some tests are straightforward; for example, a histogram of $A_{\ell m}$ amplitudes normalized to a best-fit smooth C_ℓ spectrum might show statistically significant departures from a gaussian distribution. These discreteness effects are qualititively different from (and have a more distinctively quantum character than) other possible new effects, for example string-theory-inspired predictions [58,63] of departures from standard inflationary theory for the tensor and scalar power spectra. However, if the level of discreteness is as small as $\Delta T/T \approx 10^{-10}$, and appears only as discrete levels of Sachs-Wolfe perturbation amplitude (say), plasma motions during recombination, as well as nonlinear couplings, are likely to smear out all the observable traces of primordial discreteness even on the largest scales.

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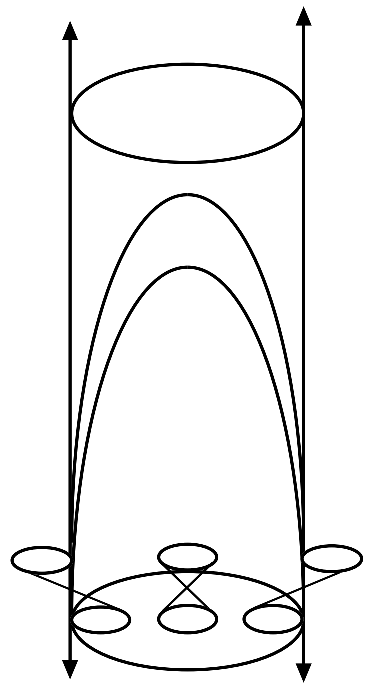

FIG. 1. Embedding diagram of an observable region of de Sitter spacetime, with one spatial dimension supressed, in proper physical units. The cylinder represents the world-sheet swept out by the event horizon (a 2-sphere) of an inertial observer at the center. Horizontal sheets correspond to the slices of constant time in Eq. (1); curved sheets represent schematically the behavior of the constant-time surfaces in Eq. (2).

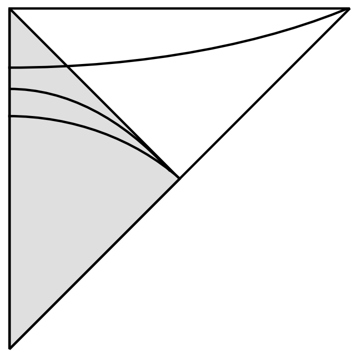

FIG. 2. Penrose diagram of the portions of de Sitter space described by the metrics, Eq. (1) and Eq. (2), showing the same spatial slicings as Fig. (1). The shaded region corresponds to the observable region shown in Fig. (1) (the whole region described by Eq. (2)).

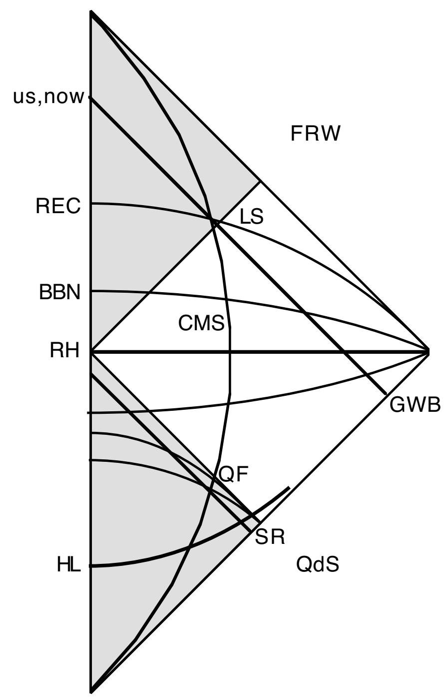

FIG. 3. Penrose diagram of an inflationary cosmology, showing the route of information flow from inflation to observable anisotropy. As usual, points represents spheres, the left hand edge represents the world line of an observer at the origin, and the extremities represent boundaries at infinity. The lower half represents a quasi- de Sitter space (QdS), the upper half represents a Friedmann-Robertson-Walker space (FRW). The join between them is the epoch of reheating (RH), and shaded regions of each show the regions with the "apparent horizon" of an observer at the origin (apparent event horizon for QdS, apparent particle horizon for FRW). Labled spacelike hypersurfaces include the recombination epoch (REC) and Big Bang Nucleosynthesis (BBN). Spacelike hypersurfaces in the QdS phase are shown for two different slicings, one appropriate for matching onto FRW and the other for holographic analysis. The intersection of our past light cone with the recombination surface is the two-sphere of the "last scattering surface" (LS) of the cosmic background radiation. The timelike trajectory of a comoving sphere (CMS) is shown, first within the inflationary event horizon, then passing outside of it, then eventually reentering the apparent particle horizon during the FRW phase. The particular CMS shown is one that enters close to recombination, and therefore is on a scale that affects the observed microwave anisotropy. Perturbations are imprinted by fluctuating quantum fields (QF) on the scale of the apparent horizon during the slow-roll period of inflation (SR). (The apparent horizon grows slightly during SR as indicated by the two closely parallel null lines, as expanded in Fig. (4).) This includes contributions from both tensor and scalar modes that are frozen in to the classical metric outside the apparent horizon. A high-frequency gravitational wave background (GWB) reaches us via direct null trajectories. The classical continuum picture breaks down for all past trajectories beyond some holographic limit (HL).

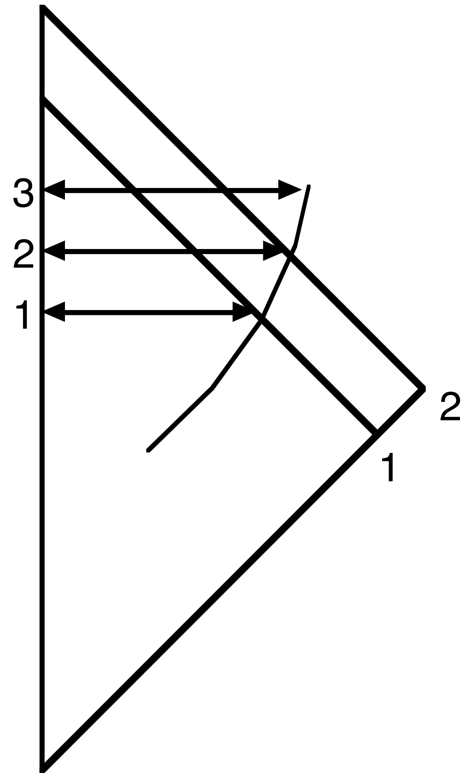

FIG. 4. A close-up view of freeze-out: the formation and "collapse" of a quantum fluctuation into a classical perturbation. It takes about one Hubble time for a perturbation on the horizon scale to freeze out. During this time, the inflaton rolls slightly and H decreases slightly; the corresponding apparent horizons are shown as 1 and 2 in the figure. Arrows indicate the physical size of a comoving mode as it passes through the horizon. By time 3, the perturbation is "outside the horizon" and the frozen-in perturbed value of the inflaton fixes the expansion rate of the background spacetime on subhorizon scales. The total increase in entropy during the interval (1,2) (that is, the difference in entropy between the two triangles) is about 10^{10}, but the frozen-in perturbation represents only a small fraction of these degrees of freedom, about 10^5.